Physics A I

Answer the following questions concerning classical mechanics.

I As shown in the figure below, consider a system in which one of the endpoints of a spring is fixed to a wall and a mass point of mass m is attached to the other endpoint and the mass point oscillates in one dimensional horizontal direction (referred to as x-axis). Here, let the origin position ($x = 0$) be the position of the mass point in the equilibrium state in which the spring does not stretch nor shorten, the positive direction be the direction along which the spring is stretched, and the spring constant be k.

(1) Show the equation of motion of the system in which the mass point described above moves against a resistance force proportional to the speed (here, let the proportionality constant be α). Simplify the equation of motion, by using angular frequency $\omega = \sqrt{k/m}$, and by rewriting the proportionality constant of the force as $\lambda = \frac{\alpha}{2m}$.

(2) The solution of the equation of motion obtained in Question (1) can be generally expressed as $x = \beta e^{\gamma t}$ with a constant γ. The β is a constant determined by conditions of the motion. Find an equation for γ. With an attention to the magnitude relationship between λ and ω ($\lambda<\omega$ or $\lambda>\omega$), it can be seen that there exist different solutions for each case ($\lambda<\omega$ or $\lambda> \omega$) which show different physical behaviors. Find general solutions for these two cases. Here, it is not necessary to consider the case of $\lambda = \omega$.

(3) Show schematic diagrams representing the time evolution of the two general solutions obtained in Question (2) and explain the difference in their physical behaviors. Here, the mass point starts its motion from a point with a small distance from the equilibrium position ($x = 0$) in the positive direction. Here, if the physical difference between the time variations of the two solutions is explained qualitatively, the quantitative differences in the angular velocity ω and the constant β may be ignored.

(4) Consider a case that an external oscillating force with a frequency of Ω acts on the mass point of the spring model discussed above. To obtain the solution for the case of $\lambda<\omega$, let the external force

be $F = e^{i\Omega t}$ and show the equation of motion.

(5) Find a general solution of the equation of motion obtained in Question (4), and describe the behavior of the solution when sufficient time has passed.

Physics A II

Answer the following questions related to electromagnetism

II

(1) Draw magnetic field lines around a bar magnet, and a circular electric current loop, respectively. The directions of the electric current, magnetization, and magnetic field lines should be clearly written.

(2) Consider a circular electric current loop of radius r [m]. The area of the circle is $S = \pi r^2$, and the permeability of vacuum is μ_0 , Find the magnetic moment M induced by the circular current I using Ampere's theorem of equivalent magnetic shell.

(3) Consider an electron of charge $-e$ [C] and mass m [kg] moving along a circular orbit of a radius r [m] around an atomic nucleus with a velocity of v, as shown in FIGURE 1. Find the magnitude of the circular current I corresponding to the electron motion, and rewrite the expression of the magnetic moment M obtained in Question (2).

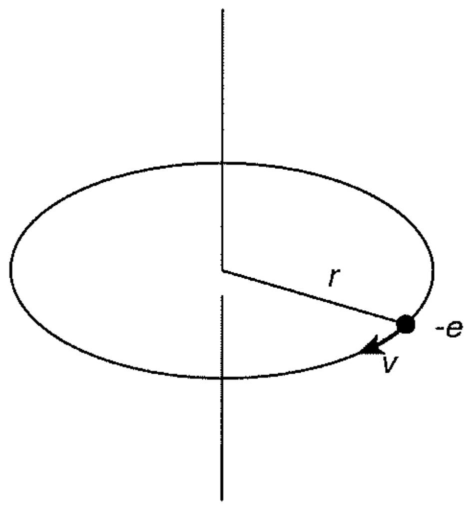

FIGURE 1

(4) Orbital angular momentum of an electron in a stationary state circulating around an atomic nucleus can take the values of only integer multiple of $h/2\pi$ (here, h=6.624x10^{-34} Js is Planck's constant). When this integer is l (which is called azimuthal quantum number), the angular momentum can be expressed as;

$$P_l = mrv = l\frac{h}{2\pi}$$

Rewrite the expression of the magnetic moment obtained in Question (3) in terms of l.

(5) The expression of the magnetic moment obtained in Question (4) can be rewritten as $\mu_B\, l$. Find

the expression of μ_B, and calculate the specific value of μ_B by using the values of the mass-to-charge ratio e/m=1.759×10^{11} [C/kg] and the permeability of vacuum $\mu_0=4\pi\cdot10^{-7}$. The μ_B is the minimum unit of magnetic moments existing in nature. What is the name of μ_B?