Physics A I

Answer the following questions on classical mechanics.

I

As shown in Figure 1, a solid cylinder of a radius a, mass m, and moment of inertia I around its center rolls without slipping on the inner surface of a hollow cylinder of a radius R. The system is assumed to be uniform along the axis O of the hollow cylinder and thus the motion of the solid cylinder can be described on a two-dimensional plane. Gravity is applied along the vertically downward direction and the acceleration of gravity is set to g. Air resistance is ignored.

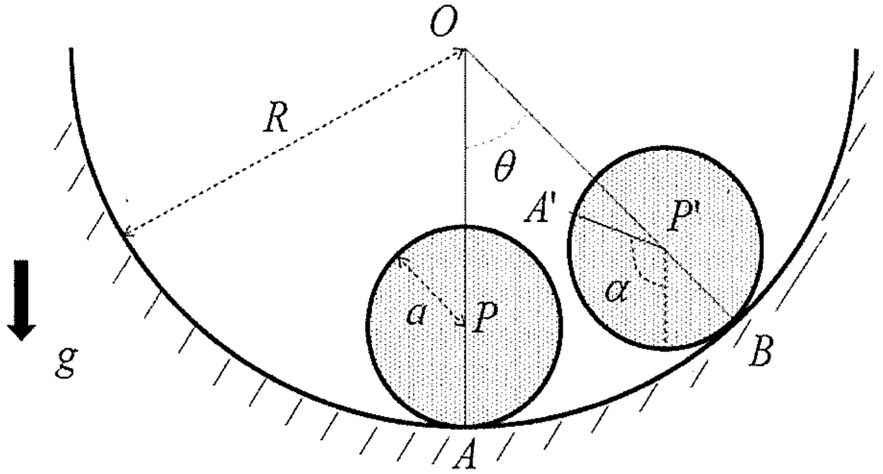

Figure 1

(1) When the center-of-mass of the solid cylinder moves from a point P to P', the contact point A between the solid cylinder and the hollow cylinder moves to a point A'. The angles between the segments OP and OP' and between the segment $P'A'$ and the vertically downward line are set to θ and α, respectively. Show that the derivatives of these two angle variables satisfy the relation of $(R-a)\dot{\theta} = a\dot{\alpha}$.

(2) Consider the moment that the center-of-mass of the solid cylinder reaches P' and investigate the motion of the solid cylinder. You only need to consider the motion along the direction perpendicular to the segment OB. Friction force at the contact point B is set to F. Obtain equations of motion as functions of θ and α for both the center-of-mass of the solid cylinder and the rotation of the solid cylinder around the center-of-mass.

(3) Using the results of the questions (1) and (2) and then eliminating θ and F, derive an

equation of motion with respect to θ.

(4) Assuming that θ is sufficiently small in the equation derived in the question (3), solve the equation and explain that its motion is a simple harmonic motion. In addition, obtain a period of the oscillation.

Physics A II

Answer the following questions concerning electromagnetism.

II. Consider electromagnetic waves propagating in a uniform and transparent medium.

(1) Write down the four Maxwell's equations in differential form. Electric and magnetic fields are $\boldsymbol{E}(x, y, z, t)$ and $\boldsymbol{H}(x, y, z, t)$, respectively. Use the MKS system of units. x, y, z are the space coordinates, and t is the time. The permittivity and permeability of the medium are ϵ and μ, respectively. Both the charge and current densities are zero.

(2) When the electromagnetic wave is a plane wave propagating along the z-axis, $E_z = 0$ and $H_z = 0$. Prove the following relationships between the electric field in the x-axis E_x and the magnetic field in the y-axis H_y in this condition,

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \tag{1}$$

$$\frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t}. \tag{2}$$

(3) E_x and H_y of a plane electromagnetic wave propagating in the z-axis direction can be expressed as follows,

$$E_x = F(z - vt) + G(z + vt) \tag{3}$$

$$H_y = \sqrt{\frac{\epsilon}{\mu}} \left\{ F(z - vt) - G(z + vt) \right\}. \tag{4}$$

Here, $v = 1/\sqrt{\epsilon\mu}$, and $F(z)$ and $G(z)$ are arbitrary differentiable functions. Prove that the E_x and H_y always satisfy the equations (1) and (2). In addition, for the plane electromagnetic wave propagating in the positive direction along the z-axis, prove $H_y = \sqrt{\frac{\epsilon}{\mu}} E_x$.

(4) Assume that the electromagnetic wave linearly polarized in the x-direction propagates in a medium with the permittivity ϵ and the permeability μ in the positive direction along the z-axis. The electro magnetic wave is incident normally on a medium with the permittivity ϵ' and the permeability μ'. Express the reflected electric field $E_x^{(r)}$ and the transmitted electric field $E_x^{(t)}$ by using the incident electric field E_x. Use the boundary conditions of electric and magnetic fields.

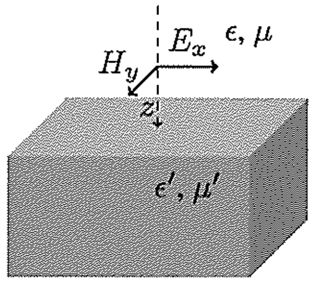

Figure 1