Physics A I

Answer the following questions on classical mechanics.

I

Consider the motion of an object of mass m on a belt conveyor with velocity V as shown in Fig. 1. Here the belt conveyor is connected on the ground and the belt is moving horizontally. When the object was gently put on the belt conveyor at time $t=0$, it first slipped on the belt conveyor. Then, it was accelerated and began to move together with the belt conveyor at time $t=t_0$. Answer the following questions using V, m, μ, or g, where μ is the dynamic friction coefficient between the object and the belt conveyor, and g is the gravity acceleration, respectively.

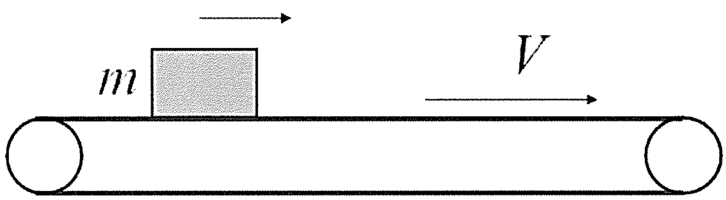

Figure 1

(1) Calculate t_0.

(2) How long does the object travel against the ground from $t=0$ to t_0?

In addition to the belt conveyor in the previous problems, a spring with a spring constant k, which can only change the length horizontally, is connected on the ground as shown in Fig. 2(a). A binding connector, which catches the object after contact, is attached to the spring, whose mass is negligible. After the object touched the binding connector, it started to slip on the belt conveyor, and then showed a periodic horizontal motion with amplitude A as shown in Fig. 2(b). Here, the speed of the belt conveyor is much faster than the object. Assume that the object and the binding connector moved together after the contact, and that there is no frictional resistance of the spring and the binding connector. Answer the following questions using V, m, μ, g, k, A, or l, where l is an equilibrium length of a spring.

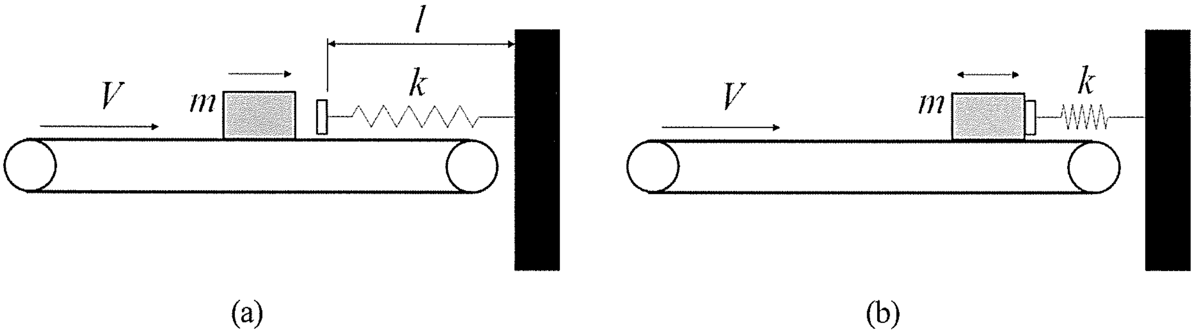

Figure 2

(3) Calculate the length of the spring at the center of the periodic motion.

(4) Calculate the period of the periodic motion of the object.

(5) Calculate the maximum speed of the object against the ground during the periodic motion.

Physics A II

Answer the following questions concerning electromagnetism.

II.

Use the MKSA system of units for all of the following questions. Let ε_0 be the vacuum permittivity.

(1) Suppose that there is a point charge Q in vacuum. Write down the potential $\phi(r)$ at the position with the distance r away from the point charge.

(2) Consider the following questions in the xyz-coordinate system as shown in Fig. 1. Assume that there are a conductor surface extending infinitely on the xy-plane and a point charge $+Q$ at the point $(0,\ 0,\ d)$ where $d > 0$ as given in the figure. The region $z > 0$ is in vacuum. The electric potential can be evaluated by assuming that there is a virtual charge $-Q$ at the position symmetric with respect to the conductor surface. Write down the electric potential $\phi(x,\ y,\ z)$ in the region $z > 0$.

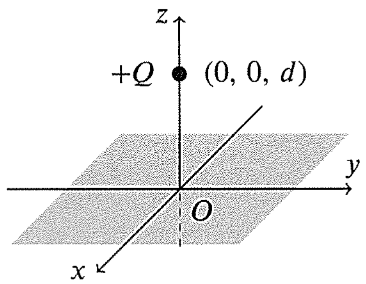

Fig. 1

(3) Calculate each component of electric field $\vec{E}(x,\ y,\ z)$, namely, $E_x(x,\ y,\ z)$, $E_y(x,\ y,\ z)$, and $E_z(x,\ y,\ z)$, in the region $z > 0$.

(4) Let the electric field on the conductor surface be $E_{z0}(x,\ y)$ and the charge density induced on the conductor surface be $\sigma(x,\ y)$, as given in Fig. 2. In this case, the boundary condition between the electric field $E_{z0}(x,\ y)$ and the charge density $\sigma(x,\ y)$ can be determined by applying one of the Maxwell's equations to the region surrounding a part of the conductor surface, such as the rectangular region shown in Fig. 2, and obtained as $\sigma(x,\ y) = \varepsilon_0 E_{z0}(x,\ y)$. Now, write down the charge density $\sigma(x,\ y)$ induced on the conductor surface.

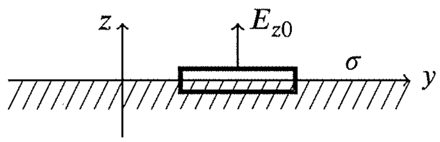

Fig. 2

(5) By integrating the charge density $\sigma(x,\ y)$, show that the total charge induced on the conductor surface is equal to $-Q$.