Physics AI

I Answer the following questions on classical mechanics.

As shown in the figure below, three point masses, which move horizontally without friction, are connected by springs, which expands or shrinks along a circle with a constant radius R. Point masses 1, 2, and 3 have mass m_1, m_2, and m_3, respectively, and these springs are identical with a spring constant k, and mass of the springs is negligible. Point masses 1, 2, and 3 initially stay at equilibrium positions A, B, and C, respectively, and positions A, B, and C equally separate the circle into three parts. The rotation angles from the equilibrium positions for point masses 1, 2, and 3 are expressed as ϕ_1, ϕ_2, and ϕ_3, respectively, and restoring force proportional to the displacement from the equilibrium length of the springs acts. Here, counter-clockwise direction is defined as positive angle, and the unit of the angle is radian. Answer the following questions, assuming that the displacement of each point mass from its equilibrium position is negligible compared with the length of the springs and the circumference of the circle ($0 \leq |\phi_1|, |\phi_2|, |\phi_3| \ll 2\pi/3$).

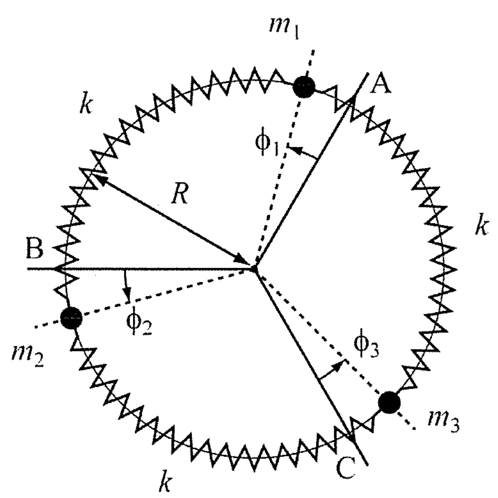

Figure 1

(1) Express the force F_1 that acts on point mass 1 using k, R, ϕ_1, ϕ_2, ϕ_3.

(2) Derive the following differential equations for the angles ϕ_1, ϕ_2, ϕ_3 on the basis of equations of motion for three point masses at time t,

$$\frac{d^2\phi_1}{dt^2} = \frac{k}{m_1}(-2\phi_1 + \phi_2 + \phi_3),$$
$$\frac{d^2\phi_2}{dt^2} = \frac{k}{m_2}(\phi_1 - 2\phi_2 + \phi_3),$$
$$\frac{d^2\phi_3}{dt^2} = \frac{k}{m_3}(\phi_1 + \phi_2 - 2\phi_3).$$

(3) Verify that the sum of the angular momentum for three point masses is conserved by using the equations in question (2).

Using the equations in question (2), answer the following questions assuming that $m_1 = m_2 = m$ and $m_3 = 2m$. Use $\omega_0 = \sqrt{k/m}$, if necessary.

(4) Verify that the oscillation frequency is given by $f = 1/2\pi \times \sqrt{3k/m}$, in the case where the point mass 3 is at rest and mass 1 and mass 2 are oscillating at the same frequency as simple harmonic oscillator.

(5) Derive the oscillation frequency in the case where all the three point masses are oscillating at the same frequency with finite amplitudes as simple harmonic oscillator.

Physics A Ⅱ

Ⅱ Answer the following questions on electromagnetism.

Consider a magnetic field induced by a static electric current I along a conducting wire in a vacuum. Use the MKSA units and the magnetic permeability of vacuum is given by μ_0. Note that the diameter of the conducting wire can be ignored.

(1) When a current I is passing through a straight infinite conducting wire, calculate the amplitude of the magnetic flux density at the point whose distance to the conducting wire is L. Note that a line integral of a magnetic flux density around some closed curve C can be expressed as $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$, using the total current I passing through a surface enclosed by C (Ampère's circuital law).

As shown in Fig. 1, a current I flows along a straight finite conducting wire XY from Y to X. Consider a magnetic flux density $\vec{B}$ at the point P, whose position is defined by $\angle\mathrm{PXY} = \theta_1$ and $\angle\mathrm{PYX} = \theta_2$, where a distance OP from conducting wire XY is described by L.

(2a) Now we define point Z on the conducting wire XY, where $\mathrm{OZ} = s$ and $\angle\mathrm{PZX} = \theta$ as shown in Fig. 1. Verify that the amplitude of the magnetic flux density $d\vec{B}$ at point P generated by a short current segment $Id\vec{s}$ at point Z is given as $dB = \dfrac{\mu_0 I}{4\pi L}\sin\theta d\theta$. You can use Biot–Savart law, where a magnetic flux density $d\vec{B}$ at $\vec{r}$ generated by a short current segment $Id\vec{s}$ can be expressed as $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{s}\times\vec{r}}{|\vec{r}|^3}$ as shown in Fig. 2.

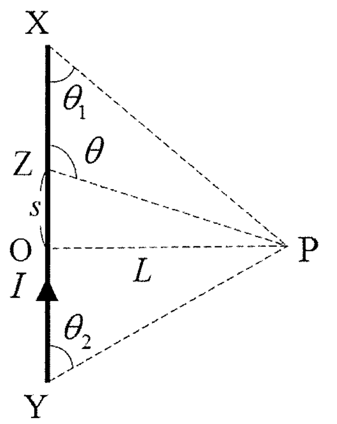

Figure 1

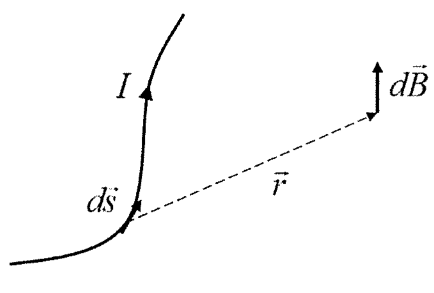

Figure 2

(2b) Calculate and express the amplitude of the magnetic flux density $\vec{B}$ at point P generated by the finite conducting wire XY using I, L, θ_1, and θ_2.

(3) Verify that the amplitude of the magnetic flux density at center O of the square circuit in Fig. 3 is given as $B = \frac{2\sqrt{2}\mu_0 I}{\pi D}$, when a current I flows along the square circuit whose side length is D.

(4) When a current I flows along a regular n-sided polygon circuit inscribed to a circle with a diameter R, calculate the amplitude of the magnetic flux density induced at center O in Fig. 4. Then calculate the amplitude of the magnetic flux density at the center of a circular loop whose diameter is R by taking infinite limit of n. Note that $\lim_{x\to\infty} x \tan\frac{1}{x} = 1$.

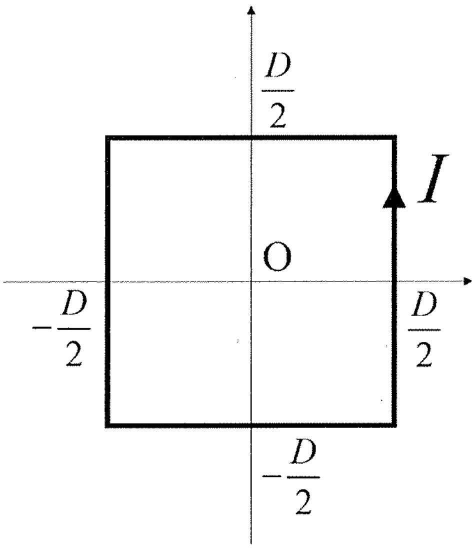

Figure 3

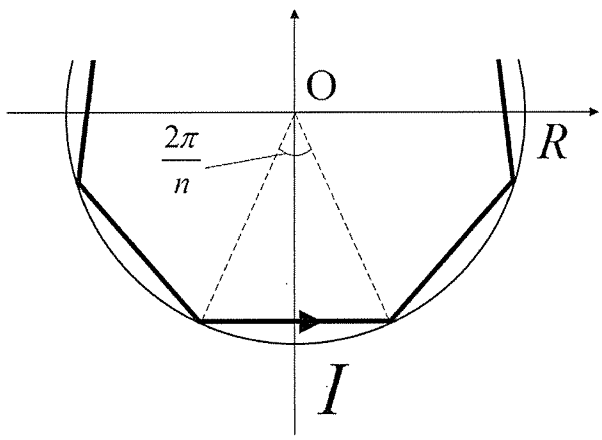

Figure 4