Physics A

[Total 1 page]

Answer the following questions on classical mechanics.

An object of mass m is dropped from a high position above the ground with an initial velocity of zero at time $t = 0$. The object is assumed to experience air resistance proportional to its velocity as it falls, and its proportional constant is *k*. Let the *z*-axis be vertical downward. Let the vertical downward velocity and acceleration be v and $\frac{\mathrm{d}v}{\mathrm{d}t}$, respectively. It is assumed that this object continues to fall and is subject to no forces other than gravity and air resistance. The gravitational acceleration is assumed to be constant and denoted by g. Answer the following questions.

(1) Write down the equation of motion for this object.

(2) Calculate the velocity v_∞ at time $t \to \infty$ under the condition that air resistance and gravity are balanced.

(3) Solve the equation of motion and calculate the velocity $v(t)$ of the object at time t with the initial velocity as $v(0) = 0$.

(4) Show that the velocity $v(t)$ can be approximated as

$$v(t) = gt$$

in the vicinity of time $t = 0$.

(5) Draw a schematic graph of the velocity $v(t)$ with time t on the horizontal axis.