【Total 2 pages】

Answer the following questions on classical mechanics. For all the following questions, we assume that spring forces obey Hooke's law and that the mass of springs and frictional force from the floor are negligible.

First, a point mass with mass M and two springs with natural length l and spring constant k are connected as shown in Fig. 1, and the ends are fixed at two walls separated by $2l$ on a horizontal floor. Let $x(t)$ be the displacements of the point mass from its equilibrium position at time t, and consider one-dimensional oscillation along the horizontal direction. For the time derivatives of position, use abbreviations such as $\dot{x}(t) = dx/dt$ and $\ddot{x}(t) = d^2x/dt^2$.

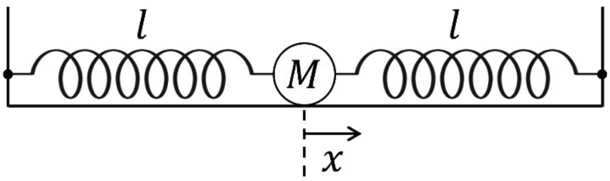

Figure 1

I Write down the equation of motion for the point mass.

II Solve the equation in question I under the following initial conditions: $x(0) = x_0$ and $\dot{x}(0) = 0$.

Then, two point masses 1 and 2 with mass m and three springs with natural length l and spring constant k are connected as shown in Fig. 2, and the ends are fixed at two walls separated by $3l$ on a horizontal floor. Let $x_1(t)$ and $x_2(t)$ be the displacements of point masses 1 and 2, respectively, from their respective equilibrium positions at time t, and consider one-dimensional oscillation along the horizontal direction.

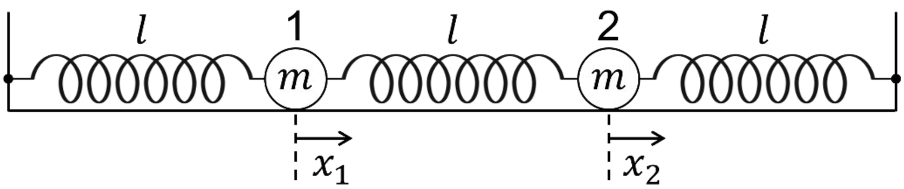

Figure 2

III Write down the equations of motion for point masses 1 and 2.

IV Let $X(t) = \{x_1(t) + x_2(t)\}/2$ and $Y(t) = \{x_1(t) - x_2(t)\}/2$, which correspond to the center-of-mass position and relative position of the point masses, respectively. Show the equations of motion with respect to $X(t)$ and $Y(t)$ and then solve these under the following initial conditions:
$x_1(0) = x_{10},\ x_2(0) = x_{20}$, and $\dot{x}_1(0) = \dot{x}_2(0) = 0$.

V Show that when $x_{10} = x_{20}$, only the oscillation of the center-of-mass position $X(t)$ appears in the solution obtained in question IV. Under this condition, the oscillation can be viewed as that of a single point mass depicted in Fig. 1. Show what the relationship is between m and M in the case where the angular frequency of this center-of-mass oscillation coincides with the angular frequency obtained in question II.

(The end)

【Total 1 page】

Answer the following questions on electromagnetism.

The speed of light is c (= 3 × 10^8 m s^{-1}). Use the MKSA unit system as the unit system.

I When the electric field is $\boldsymbol{E}(x, y, z, t)$ and the magnetic flux density is $\boldsymbol{B}(x, y, z, t)$, write Maxwell's equations in vacuum where neither charge nor current exists. Here, x, y, and z represent the coordinates of the space in the Cartesian coordinate system and t is the time. Use ε_0 for the permittivity and μ_0 for permeability.

II Derive an equation for the energy conservation law for the electromagnetic fields using the Maxwell's equations. The energy density of electromagnetic fields u is $\frac{1}{2}\varepsilon_0\boldsymbol{E}\cdot\boldsymbol{E}+\frac{1}{2\mu_0}\boldsymbol{B}\cdot\boldsymbol{B}$ and the energy flux of electromagnetic fields $\boldsymbol{S}$ is $\frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}$. The derivation process should be shown.

III Consider the case where $\boldsymbol{E}$ and $\boldsymbol{B}$ are independent of x and y and are functions of z and t only. When $E_x = E_0\sin(kz-\omega t)$, $E_y = 0$, $E_z = 0$, $B_x = 0$, $B_z = 0$, express u using E_x and ε_0, and express $|\boldsymbol{S}|$ using E_x, ε_0, and c. Here, E_0 is the amplitude, k is the wave number, ω is the angular frequency, and the subscripts in E and B represent the x, y, and z components. If necessary, $\omega = kc$ and $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ can be used.

IV When u = 36 J (Joule) m^{-3}, calculate E_x and B_y. ε_0 is 9 × 10^{-12} F (Farad) m^{-1}. Answer with one significant digit.

(The end)