Physics A I

I Answer the following questions on classical mechanics.

Consider one dimensional vibrational motion of a point mass with mass m that is connected to the wall by a spring with spring constant k. As shown in Fig. 1, the position of the point mass from the natural length at time t is denoted by $x(t)$. Ignore the mass of the spring, gravity, and frictional force from the flat floor. Use the following notations for time derivatives of the position: $\dot{x} = dx/dt$, $\ddot{x} = d^2x/dt^2$.

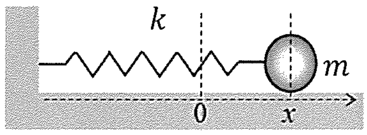

Figure 1

(1) Derive the equation of motion of the point mass under the force only from the spring.

(2) Solve the equation of motion in the question (1) under the following initial condition at time 0, $x = x_0$ and $\dot{x} = 0$, and answer the period of the vibrational motion. Then, calculate the kinetic, potential, and total energies of the vibrational motion at time t.

(3) Consider the vibrational motion of the point mass under the force from the spring and a resistive force proportional to the velocity $-\eta\dot{x}$, where η is a positive constant. Then, write the equation of motion of this vibrational motion, and verify that the equation of motion is described as $\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$. Express ω_0 and γ using m, k and η. Note that ω_0 and γ are positive and called as natural frequency and damping rate, respectively.

In the following questions, consider the situation where a time dependent external force $F(t,\omega) = F_0 exp(-i\omega t)$ is applied along the x axis to the point mass in the question (3).

(4) Derive the equation of motion of this vibrational motion. Then, solve the equation of motion and express the complex amplitude $A(\omega)$ with ω_0 and γ, assuming that the solution of this equation is expressed as the following form: $x(t,\omega) = A(\omega)exp(-i\omega t)$.

(5) When the frequency ω is around ω_0 ($\omega \approx \omega_0$), the following approximation is valid: $\omega_0^2 - \omega^2 = (\omega_0 + \omega)(\omega_0 - \omega) \approx 2\omega_0(\omega_0 - \omega)$. By using this approximation, verify that the complex amplitude $A(\omega)$ derived in the question (4) is expressed as $A(\omega) = \frac{F_0}{2m\omega_0}\frac{1}{\omega_0 - \omega - i\gamma}$. This complex amplitude is also expressed as the form $A(\omega) = |A(\omega)|exp(i\theta(\omega))$. Then, answer the absolute value $|A(\omega)|$ and verify that the phase $\theta(\omega)$ satisfies the following relation: $\tan\theta(\omega) = \frac{\gamma}{\omega_0 - \omega}$. Finally, verify that phase of the vibrational motion of the point mass $x(t,\omega)$ is deviated by $\theta(\omega)$ from the externally applied force $F(t,\omega)$.

(6) Choose the most appropriate sketch of $|A(\omega)|$ and $\theta(\omega)$ from the Figures 2(a)-(h) for both of the cases with the resistive force ($\eta > 0$) and without the resistive force ($\eta = 0$).

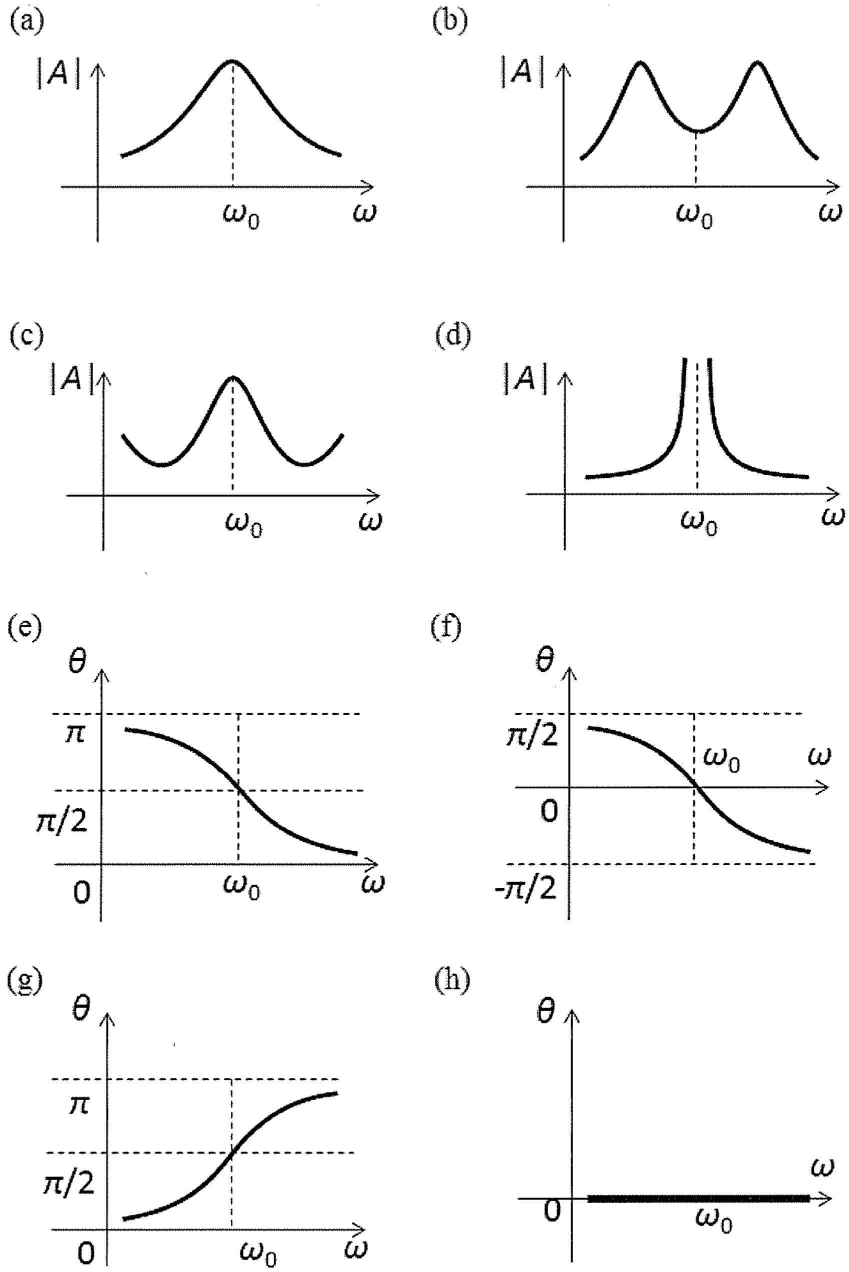
(a)
|A|
ω0
ω
(b)
|A|
ω0
ω
(c)
|A|
ω0
ω
(d)
|A|
ω0
ω
(e)
θ
π
π/2
0
ω0
ω
(f)
θ
π/2
ω0
ω
0
-π/2
(g)
θ
π
π/2
0
ω0
ω
(h)
θ
ω
0
ω0

Figure 2

Physics A II

II Answer the following questions on electromagnetism.

Use the MKSA units and consider a particle with electric charge q and mass m moving in the electromagnetic field. Here, the charged particle moves in a vacuum under no gravity.

(1) When a charged particle moves in the electromagnetic field, the force $\boldsymbol{F}$ acting on the charged particle is given by $\boldsymbol{F} = q\boldsymbol{E} + q\frac{d\boldsymbol{r}}{dt} \times \boldsymbol{B}$ where the position vector $\boldsymbol{r} = (x, y, z)$. Here, $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic flux density, and t is time. Derive the x, y, and z components of the equations of motion for the charged particle under the uniform electric field $\boldsymbol{E} = (E_x, E_y, E_z)$ and magnetic flux density $\boldsymbol{B} = (B_x, B_y, B_z)$.

(2) When $E_x = E_y = E_z = 0$, $B_x = B_y = 0$, and $B_z = B_0$, solve the equations of motion obtained in question (1) for x, y, and z components. Here, initial conditions at $t = 0$ are $\boldsymbol{r} = (0,0,0)$ and $\frac{d\boldsymbol{r}}{dt} = (v_0, 0, 0)$ where $v_0 > 0$. If necessary, you can use that the general solution of the differential equation $\frac{d^2 f}{dt^2} = -\omega^2 f$ is given as $f(t) = C_1 \sin \omega t + C_2 \cos \omega t$. Here, C_1 and C_2 are constants and ω is a real number.

(3) Using the solution obtained in question (2), prove that the trajectory of the charged particle is a closed circle and also calculate its radius and coordinate of the center.

(4) It is assumed that the charged particle is ejected at time $t = 0$ from a position $\boldsymbol{r} = (0,0,0)$ with a velocity $\frac{d\boldsymbol{r}}{dt} = (v_0, 0, 0)$ where $v_0 > 0$. The electric field $E_y = E_0$, $E_x = E_z = 0$ and magnetic field $B_x = B_y = B_z = 0$ are applied until $t = t_0$. After $t = t_0$, these fields are not applied ($E_x = E_y = E_z = 0$ and $B_x = B_y = B_z = 0$). Answer positions of y and z when the particle reaches the yz-plane at $x = x_0$. Note that x_0 is a positive real number.