Physics A I

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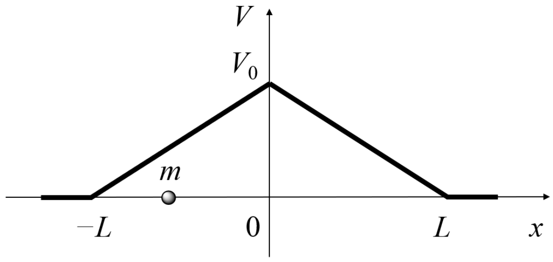
Answer the following questions on classical mechanics.

I-a

We consider the motion of a charged particle under the electric potential,

$$V(x) = \begin{cases} V_0\left(1 - \dfrac{|x|}{L}\right) & (-L \le x \le L) \\ 0 & (x < -L,\ L < x) \end{cases},$$



Figure

where $V_0 > 0$ and $L > 0$, as shown in Figure. This charged particle with mass m and charge $-q$ moves under the force only with electric potential $(m > 0,\ q > 0)$. The total energy of this charged particle is given by $-W_0\ (0 < W_0 < qV_0)$ and this charged particle shows periodic motion along the x-axis within $-L < x < L$. Answer the following questions.

(1) Write down the equation representing the energy conservation rule of the charged particle. The velocity of the particle should be written as v.

(2) Calculate the range of the periodic motion of the charged particle in the x-axis.

(3) Show a schematic diagram of a trajectory representing the motion of the charged particle with the momentum in a vertical axis and the position in a horizontal axis. Answer the values at which the trajectory of the motion intersects the vertical and horizontal axes.

(4) Write down the equation of motion of the charged particle moving to the positive direction in the range of $x < 0$, and then solve the equation of motion. At $t = 0$, this particle is located at the most distant position from $x = 0$ in the range of $x < 0$ which was solved in question (2) and its velocity is zero.

(5) Calculate the period of the periodic motion of the charged particle.

Physics A II

【Total 1 page】

Answer the following questions on electromagnetism.

II-a

An electric charge is distributed in a sphere of radius R with uniform density ρ, as shown in Figure. Answer the following questions concerning the strength of the electric field $E(r)$ created by this charge at a distance r from the sphere's center. Let the circular constant be π. If necessary, Gauss's law may be used.

Gauss's law

The following equation holds for any closed surface S.

$$\int_S \{\boldsymbol{E}(\boldsymbol{r})\cdot\boldsymbol{n}(\boldsymbol{r})\}\,\mathrm{d}S = \frac{1}{\varepsilon_0}\int_V \rho(\boldsymbol{r})\,\mathrm{d}V,$$

where $\boldsymbol{E}(\boldsymbol{r})$ is the electric field at point $\boldsymbol{r}$ on the surface S, $\boldsymbol{n}(\boldsymbol{r})$ is the unit vector perpendicular to the surface S and outward at point $\boldsymbol{r}$, $\rho(\boldsymbol{r})$ is the charge density, and ε_0 is the dielectric constant in vacuum. The integral on the left-hand side is the integral over the entire surface S. The integral on the right-hand side is the integral over the whole region V bounded by the surface S.

(1) Calculate the electric charge Q contained in the sphere of radius R.

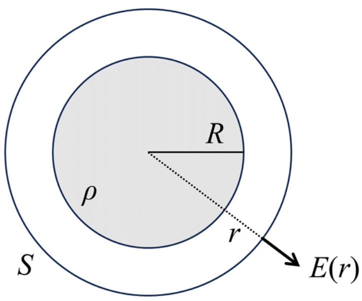
(2) Calculate the strength of the electric field $E(r)$ for $r > R$.

(3) Next we want to find the strength of the electric field $E(r)$ at $r \le R$. Calculate the charge Q contained in the sphere of radius r ($\le R$).

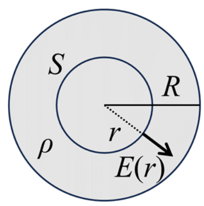
(4) Calculate the strength of the electric field $E(r)$ at $r \le R$.

(5) Draw a graph of the strength of the electric field $E(r)$ as a function of radius r. The strength of the electric field at $r = R$, $E(R)$, should be expressed using ρ, R, and ε_0 on the vertical axis.

(6) Next we consider the case where all the charges Q contained in this sphere of radius R gather at the center as a point charge. In this case, find the strength of the electric field $E(r)$ at a point at a distance r from the center of the sphere. Also, state whether this is the same or different from $E(r)$ obtained in (2) for $r > R$.



$r > R$



$r \le R$

Figure