Physics A I

【Total 2 pages】

Answer the following questions on classical mechanics.

I

As shown in Fig. 1, a small ball of mass m was attached to one end of a thread of length l, the other end of which was fixed at point O. The small ball was lifted up to point A, which is at the same height as point O, so that the thread would not loosen, and then gently released. After passing through the lowest point B, the small ball made a circular motion around a peg placed at point C, which is located at a distance of r_1 directly above point B. Assuming that a gravity acceleration is g, the mass of the thread and the size of the small ball are negligible, answer the following questions.

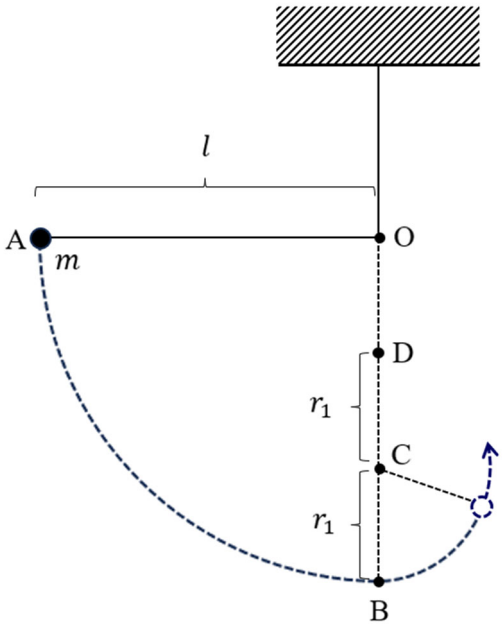

Figure 1

(1) If the small ball continues in a circular motion from point C to point D, which is directly above by r_1, calculate the velocity v_D of the small ball at point D.

(2) In order that the small ball passes through point D without any loosening of the thread, the tension T_D of the thread from the small ball to point C must be $T_D \geq 0$ at point D. Write down the vertical component of the force balance equation of the small ball at point D, in which v_D and T_D can be

used.

(3) Solve the maximum distance $r_1^{\max}$ between points B and C using l so that the small ball can pass through point D without the thread loosening.

As shown in Fig. 2, when the position of point C is changed to a position directly above r_2 ($r_2 > r_1^{\max}$) away from point B, the small ball stops its circular motion at the position of point E where the angle between the points $\angle OCE=\theta\ (0<\theta<\pi/2)$, and thereafter, the ball moves in parabolic motion. Answer the following questions.

(4) Express $\cos\theta$ with l and r_2, using the fact that the tension of the thread acting on the small ball at point E is zero.

(5) After passing through point E, the small ball moved in parabolic motion and hit point C. Solve the length of r_2 that satisfies this condition using l, and find $\cos\theta$.

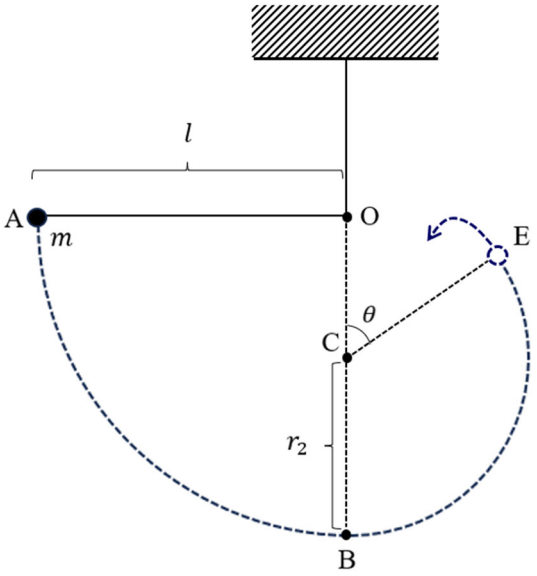

Figure 2

(The end)

Physics A II

【Total 2 pages】

Answer the following questions on electromagnetism.

II

Consider the motion of an electron of charge $-e$ and mass m in a uniform electrostatic field and a uniform static magnetic field. Use the MKSA unit system as the unit system. Assume that an electron is moving in a vacuum and that the effect of gravity is negligible.

(1) The force acting on the electron moving in the electromagnetic fields is given by $-e\mathbf{E} - e\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} \times \mathbf{B}$ using the position vector of the electron **r**. Here, **E** is the electric field vector, **B** is the magnetic flux density vector, and t is the time. Write the equation of motion for **r**.

(2) Consider the case where **E** and **B** are $\mathbf{E} = (E_x, E_y, E_z) = (-E, 0, 0)$ and $\mathbf{B} = (B_x, B_y, B_z) = (0, 0, B)$, respectively, as shown in Figure 1. The initial values at $t = 0$ are given by $\mathbf{r} = (x, y, z) = (0, 0, 0)$ and $\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \left(\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}y}{\mathrm{d}t}, \frac{\mathrm{d}z}{\mathrm{d}t}\right) = (0, 0, 0)$. Write the equations of motion for x and y in this case, respectively.

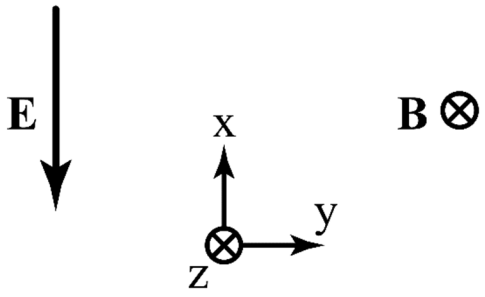

Figure 1

(3) Assuming that the position of the electron in the x direction is given by $x(t) = -\alpha\cos(\omega t) + \frac{E}{B\omega}$, express the positive real number α using e, m, E, and B. Here ω is the positive real number.

(4) Solve for the position of the electron in the y direction, $y(t)$, based on the answer to question (3).

(5) An orbit of the electron in the x-y plane is cycloid from the answers to questions (3) and (4). Cycloid is the trajectory by a point on a circle as it rolls along a straight line without slipping. Draw an electron orbit at $t \geq 0$ on your answer sheet using Figure 2 below.

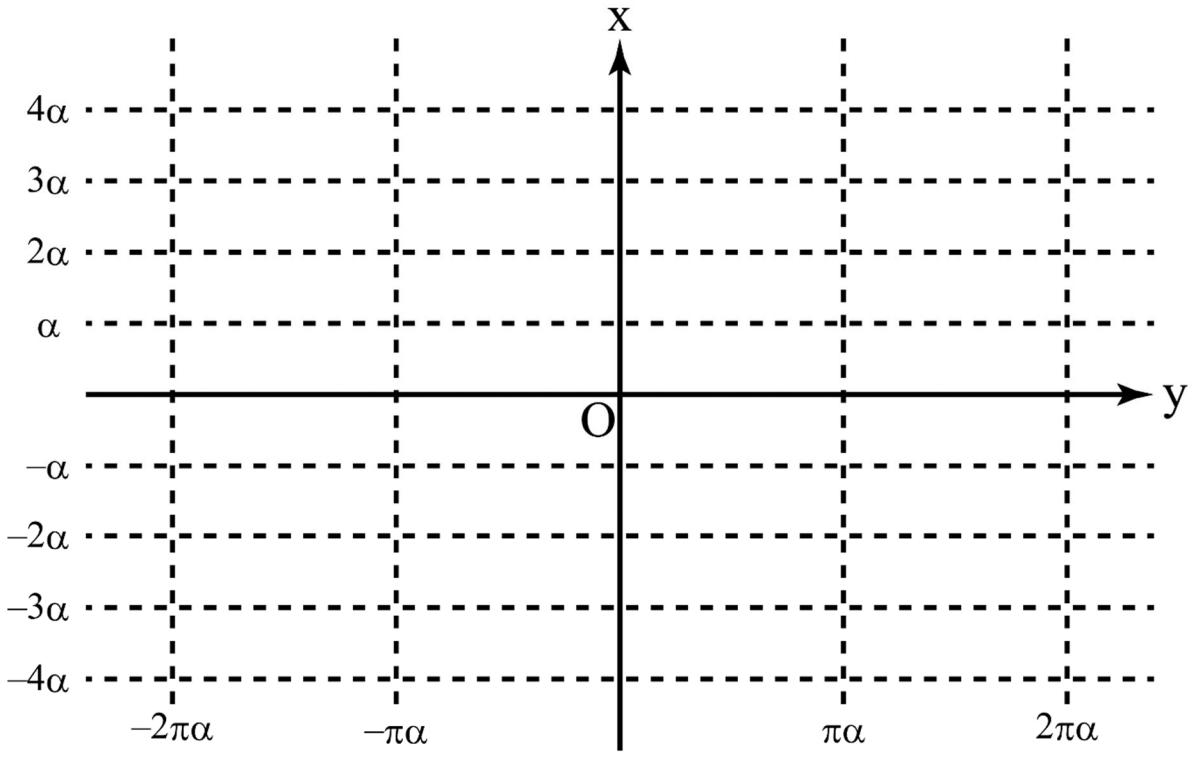

Figure 2

(6) The position of the electron oscillates with the angular frequency ω in the x direction. When B = 1.0 T (Tesla), find ω. The elementary charge is 1.6×10^{-19} C (Coulomb) and the electron mass is 9.1×10^{-31} kg. Answer with two significant digits.

(The end)